elevate education

Maths

Practice Questions

Year 11 & 12 (NSW)

Instructions

Individual, exam-style questions

The questions contained in this booklet match the style of questions that are typically asked in exams. This booklet is not however, a practice exam. Elevate's research with top students identified that top students do more practice questions than anyone else. They begin the process of testing their knowledge early in the year.

Therefore, we have provided exam-format questions that are sorted by topic so that you can answer them as you learn the information, rather than waiting until the very end of the year to complete exams.

Comments, questions?

Let us know if you need any further advice by visiting www.elevateeducation.com. You can comment on any of our material, or head to the FAQ section and ask us a question. Also, you can find us on social media so you can stay up to date on any brand new tips we release throughout the year.

Other information

Every effort has been made to ensure the accuracy of the information expressed in this booklet, but no warranty or fitness is implied. If you'd like to provide any feedback on this booklet, let us know at admin@elevateeducation.com.

Calculus

1. Differentiate $f(x) = x^2 + 5x - 4$ from first principles

2. Differentiate $\frac{x-2}{\cos(x)}$ with respect to x

3. Differentiate $e^{2x}.x^3$ with respect to x

4. Find $\int (2x+4)^5 dx$

5. Find $\int \sec^2(3x) + 1\ dx$

6. Find $\int \frac{3}{(2x-5)^2} dx$

7. Find $\int \frac{x}{x^2+4} dx$

8. Find $\int_e^{e^3} \frac{4}{x} dx$

9. Let $f(x) = x^3(2x+5)$

   I. Find $\frac{dy}{dx}$

   II. Find the x coordinates of the stationary points and determine their nature

   III. Sketch the curve y=f(x) labelling the y intercept

   IV. For what values of x is the curve concave up?

10. The following table lists the values of a function for 3 values of x

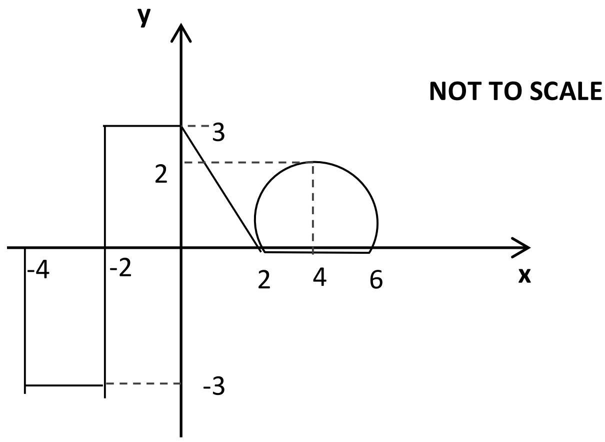
x	1.0	2.0	3.0
f(x)	-11	2	31

Use these functions to estimate $\int_1^3 f(x)dx$ by:

I. Simpson's Rule

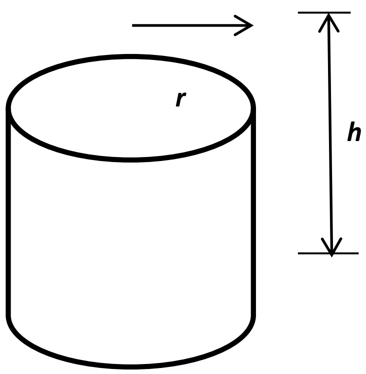
II. Trapezoidal Rule

11. The function $f(x)$ is displayed on the diagram below:



Evaluate $\int_{-2}^{6} f(x)dx$

12. A grain silo is designed in the shape of a cylinder with radius r meters and height h meters.



The volume of the tank is to be 20 cubic metres. Let A be the surface area of the tank including top and base in square meters.

I. Given $A = 2\pi r^2 + 2\pi h$. Show that $A = 2\pi r^2 + \frac{40}{\pi r}$

II. Show that A has a minimum value and find the value of r for which the minimum occurs.

13. Consider the curve given by $y = \frac{1}{2}x^4 - x^3$

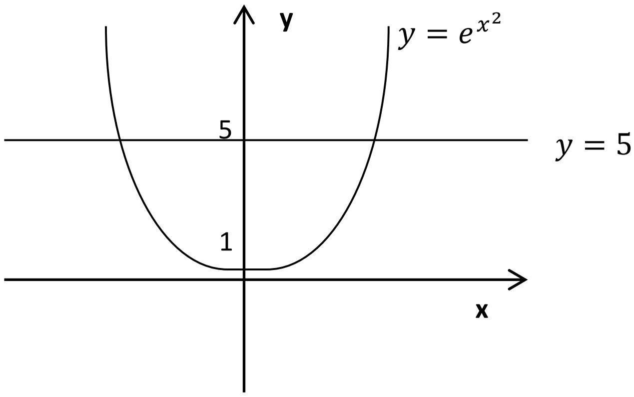
I. Find any turning points and determine their nature

II. Find any points of inflexion

III. Sketch the curve for $-2 \leq x \leq 4$ indicating where the curve crosses the x axis

IV. For what values is the curve concave down?

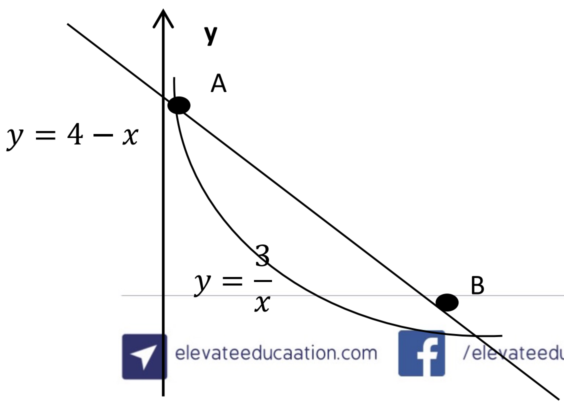
14. The region bounded by $y = e^{x^2}, y = 5$ and the y axis is rotated around the y axis to form a solid



NOT TO SCALE

Show that the volume of the solid is given by $V = \pi \int_1^5 \log_e y \, dy$

15. The diagram shows the graphs of $y = \frac{3}{x}$ and $y = 4 - x$. The graphs intersect at the points A and B as shown.



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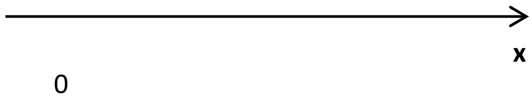


I. Find the x coordinates of the points A and B

II. Find the area of region between $y = \frac{3}{x}$ and $y = 4 - x$

Sequence and Series

1. Find the sum of the first 35 terms of the arithmetic series $4 + 9 + 14 + \cdots$.

2. A sum of $70 000 is placed in a bank account and earns 1.2% per annum, compounded annually. How much money is in the account at the end of 5 years, just after the final interest has been paid?

3. Consider the geometric series $a + ar + ar^2 + \cdots + ar^n$
   I. For what values of r does the geometric series have a limiting sum?
   II. Find a geometric series with the equation $\frac{1}{w}$ that has a limiting sum $\frac{1}{1-w}$

4. Bruna is new to social networking site MyFace. Her first profile picture receives 15 likes. Her second picture received 20 likes. As Bruna progresses, each subsequent profile picture gains 5 more likes than the one before it.
   I. How many likes will Bruna's 14^{th} picture have?
   II. How many likes will Bruna have received in total after her 14^{th} picture?
   III. How many times will Bruna upload a new profile picture before she receives over 200 likes?

5. Consider the geometric series $9 + 18x + 36x^2 + 72x^3 + \cdots$.
   I. For what values of x foes this series have a limiting sum?
   II. The limiting sum of this equation is 300. Find the value of x.

6. The number of members in an online Maths' enthusiasts club doubles every day. On day 1 there were 3 members and on day two there were 6 members.

   I. How many members were there on day 14?

   II. On which day was the number of members first greater than 1 million?

   III. The Site earns 0.5 cents per member per day. How much money did the site earn in the first 14 days? Give your answer to the nearest dollar.

7. Instagram has a zoom function with enlarges images by a factor of 1.4. In the original image of a cup of coffee, the cup is 50mm wide. After applying the zoom function once, the width is 70mm. After a second application of the zoom function, its height is 98mm.

   I. Calculate the width of the cup after the ninth application of the zoom function. Give your answer to the nearest mm.

   II. The width of the coffee cup in the image is required to be more than 300mm to reach instahfame. Starting from the original image, what is the least number of times the zoom function must be used?

8. A loan is made of $220 000 to be repaid over 40 years at 6% p.a reducible interest. Yearly, there are k regular payments made of $F. Interest is calculated and charged just before each repayment.

   I. Write down an expression for the amount owing after two repayments.

   II. Show that the amount owing after n repayments is

$$A_n = 220\ 000\alpha^n - \frac{kF(\alpha^n - 1)}{0.06}$$

   where $\alpha = 1 + \frac{0.06}{k}$

III. Calculate the amount of each repayment if the repayments are made quarterly (ie. k=4)

IV. How much would be saved over the term of the loan if monthly repayments are made instead of quarterly?

9. Borris retires with a lump sum of $400 000. The money is invested in a fund which pays interest each month at a rate of 3% per annum, and Borris receives a fixed monthly payment of $M from the fund. Hence, the amount in the fund after the first monthly payment is $(401 000 – M).

   I. Write down an expression for the amount $\$A_n$, left in the fund after n monthly payments.

   II. Borris chooses the value of M such that after there will be nothing left in the fund at the end of the 20^{th} year (240 payments). Find the value of M.

10. When Anitta began receiving pocket money she decided to deposit $15 into an online savers account at the beginning of each month. The money was invested at 6% per annum, compounded monthly.

    Let $P be the value of the investment after 360 years.

    Find P.

Probability

1. The probability that Borris will have cake for dessert is 0.67.

   What is the probability that Borris will not have cake for dessert?

2. 4 077 tickets are sold in the ∏-Day Raffle. If I've bought 3 tickets, what are my chances of winning:

   I. First prize

   II. Second prize if I have not won First prize

   III. First and second prize

3. Two Identical biased coins are tossed simultaneously and the outcome is recorded. After extensive trials, it is observed that the probability that both coins land showing heads is 0.25. What is the probability that both coins land showing tails?

4. Six students, A, B, C, D, E and F are running for school captain. To determine their order on the ballot paper, their names are written down and drawn from a bag at random.

   I. What is the probability that A is drawn first?

   II. What is the probability that the order of the names on the ballot paper is as follows:

A	B	C	D	E	F

5. Harry has five cards labelled with numbers. The numbers are 0,2,2,4,4. Two cards are selected from the deck at random without replacing the first card.

   I. What is the probability that he draws a 2, then a 4?

II. What is the probability that the sum of the numbers he draws is 6?

III. What is the probability that the second card drawn is a 4?

6. 2 yellow stones and 1 red stone are placed in a bag. A stone is randomly selected, removed and replaced by the other colour. Then a second stone is also chosen at random.

   I. Construct a tree diagram to show the probability of the outcomes.

   II. Determine the probability of both selected stones being yellow.

   III. Determine the probability of the second selected ball being yellow.

7. For April fool's day, Jo replaces 4 eggs in a box of a dozen, with rotten eggs. The other 8 are left untouched. His younger brother goes to make an omelette and selects three eggs at random. Using a tree diagram or otherwise, find the probability that:

   I. The first egg he selects is rotten

   II. All three eggs are rotten

   III. Exactly one selected egg is rotten

8. A spinner is split into 20 separate segments of the same size. Each segment has a letter printed on it with the letters Q, U, V, X, Y and Z not being used.

   I. The spinner is spun twice. What is the probability of the same letter being chosen at random twice?

   II. The spinner is spun three times. What is the probability that the letter F is chosen exactly twice?

9. Two friends, Annie and Holly are playing table-tennis together. In the first game, Annie has a 55% chance of winning.

   If Annie wins the first round, her increased confidence boosts her probability of winning the second round to 60%.

   If Annie doesn't win, her confidence drops and her probability of winning the second round decreases to 40%

   I. Draw a tree diagram for the two round sequence, labelling each branch with its probability.
   II. What is the probability Annie wins just one game?

10. To keep his laundry simple, James buys socks in three colours only. He owns 4 pairs of white socks, 3 pairs of black socks and 1 pair of purple socks. When sorting his laundry he closes his eyes and randomly picks two socks consecutively.

   I. Find the probability that both socks are white.
   II. Find the probability that one of the socks is purple.
   III. Find the probability that he selects a matching pair.

Geometry

1. On a number plane, mark the origin O and the points A(3,2) and B(5,-1)

    I. Find the gradients m_1 of OA and m_2 of AB

    II. Prove that OA Is perpendicular to AB

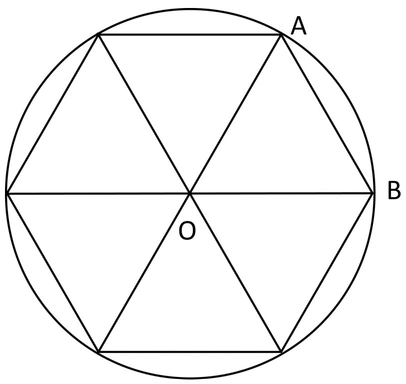
    III. Prove that OA =AB

    IV. Find the midpoint D of the interval OB

    V. Find the coordinates of the point C such that D is the midpoint of AC

    VI. What shape best describes the geometric figure OABC?

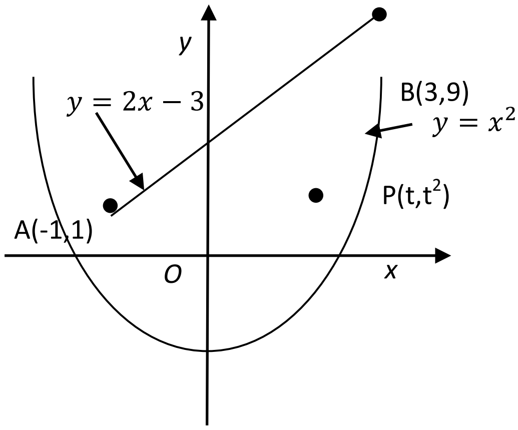
2. A regular hexagon is drawn inside a circle with centre O so that its vertices lie on the circumference, as shown in the diagram. The circle has radius 2 cm.



    I. Prove ΔOAB is equilateral.

    II. Find the area of ΔOAB and hence find the area of this hexagon. Leave your answer in surd form.

3. In the diagram A (-1,1) and B (3,9) are the points of intersection of the parabola $y=x^2$ with the line $y=2x-3$. The point $P(t,t^2)$ is a variable point on the parabola below the line.

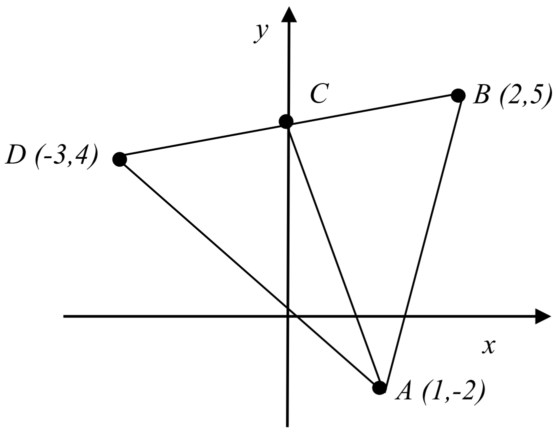


NOT TO SCALE

I. Find the area of the parabolic segment APB, i.e the area below the line and above the parabola.

II. Find the maximum area of triangle APB.

4.



NOT TO SCALE

In the diagram, A, B and D are the points (1,-2), (2,5) and (-3,4) respectively. The line BD meets the y-axis at C.

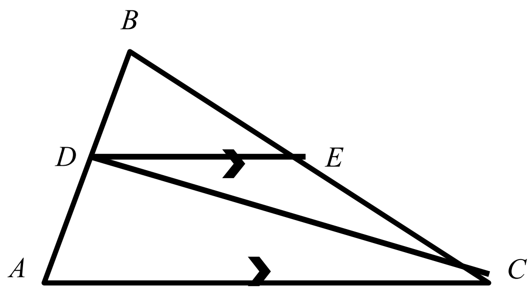
I. Show that the equation of the line DB is $x - 5y + 23 = 0$.

II. Find the coordinates of the point C.

III. Find the perpendicular distance of the point A from the line DB

IV. Hence, or otherwise find the area of triangle ADC

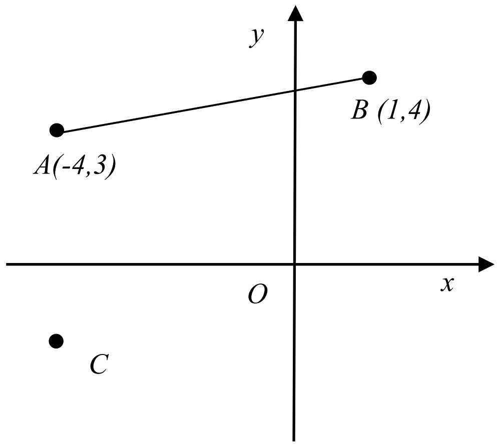
5.



In the diagram, DC bisects angle ACE and DE is parallel to AC.

Prove that the triangle DEC is Isosceles.

6.



NOT TO SCALE

I. Show the equation of the line AB is $x - 5y + 3 = 0$.

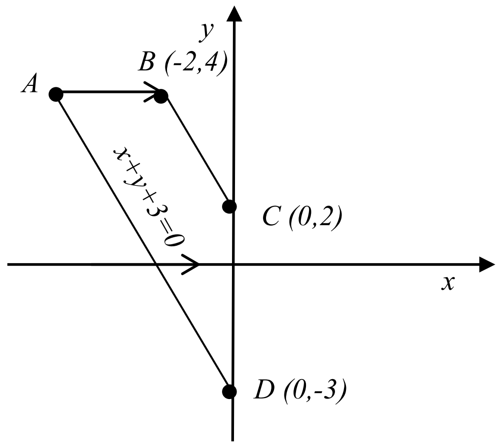
II. Show the length of AB is $\sqrt{26}$

III. Calculate the perpendicular distance between O and the line AB

IV. Calculate area of the parallelogram OBAC

V. Find the perpendicular distance from O to the line AC

7.



NOT TO SCALE

I. Show that ABCD is a trapezium by showing BC is parallel to AD

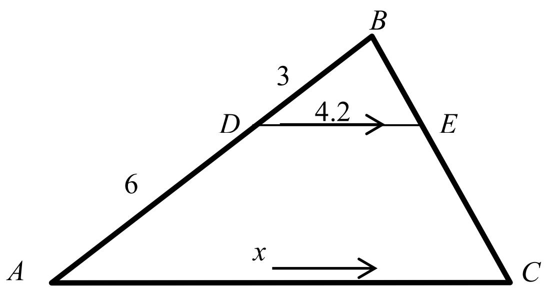
II. Find the coordinates of A

III. Find the length of BC

IV. Find the perpendicular distance from C to AD

V. Hence, or otherwise find the area of the trapezium ABCD

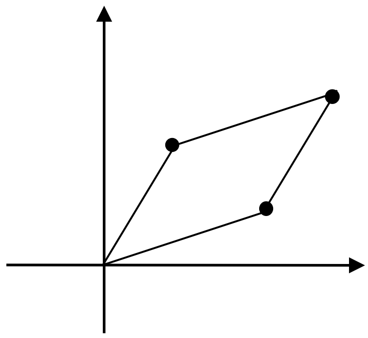
8.



In the diagram above, the line AC is parallel to the line DE, BD=3, DA=6, DE=4.2, AC=x.

Find the length AC.

9.



NOT TO SCALE

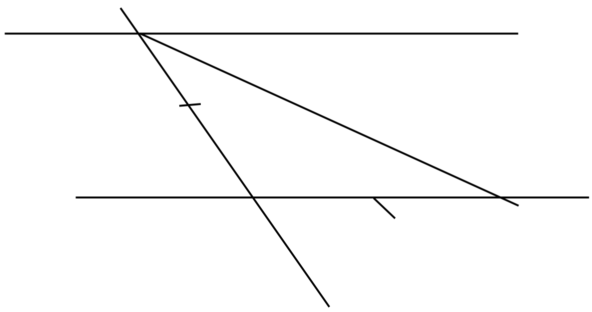
The equation of the line AB is $y = \frac{2}{3}x + 4$. The point C is (3,2).

I. Show that the line AB is parallel to the line OC

II. State why angle ABO is equivalent to angle BOC

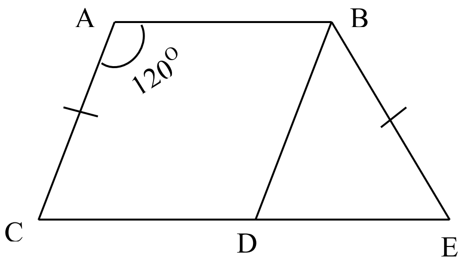
III. The line OB divides the quadrilateral OABC into two congruent triangles. Show what shape OABC makes.

10.



In the diagram, CD is parallel to AB, PR=RB, angle QRB = 52° and angle BPD = x°. Find the value of x, giving complete reasons.

11.



The Diagram shows a parallelogram ABCD with angle CAB=120°. The side CD is produced to E so that BE=AC. Prove triangle BCE is equilateral.

Rates of Change

1. The number, N, of students that received the results they wanted after being taught certain study techniques satisfies the following equation

$$N(t) = Ae^{0.75t}$$

where t is measured in days and A is a constant.

   I. When t=4, N was estimated to be 2.1×10^2. Evaluate A.

   II. The number of successful students doubles every x days, find x.

2. The number of iPhones owned by students in a high school in 2013 was 500. One year later at the same high school, 600 were owned. It is known that the number N, is given by the formula

$$N = N_o e^{kt}$$

Where N_o and k are constants and t is time in years.

   I. Find N_o and k

   II. The school has 1200 students, by what year will every student own an iPhone

3. Iron ore is extracted from an Australian mine at a rate proportional to the amount of Iron ore remaining in the mine. Hence, the amount, I, remaining after t years is given by

$$I = I_o e^{-kt}$$

Where k is constant and I_o is the initial amount of iron ore.

After 25 years, 50% of the iron ore remains.

   I. Find k

   II. How many more years will elapse before 30% of the original iron ore remains?

4. The luminous intensity I, measured in candela, of a photonic probe is given by

$$I = 10^{-8}e^{0.2L}$$

Where L is the light intensity in Lumen.

I. If the probe is at 210 Lumen, find the luminous intensity of the probe. Give your answer in scientific notation.

II. Eye damage can be caused if $I > 1.4 \times 10^{-7}$ Candela. What is the maximum light intensity which may be used without safety precautions.

III. If luminous intensity is doubled, what effect is had on the light intensity?

5. A fishery has a school of 350 bass. The number of bass, N(t), after t months is given by

$$N(t) = 350e^{kt}$$

I. After 2 years, there are 500 bass. Show that k=0.0149 to 3 significant figures.

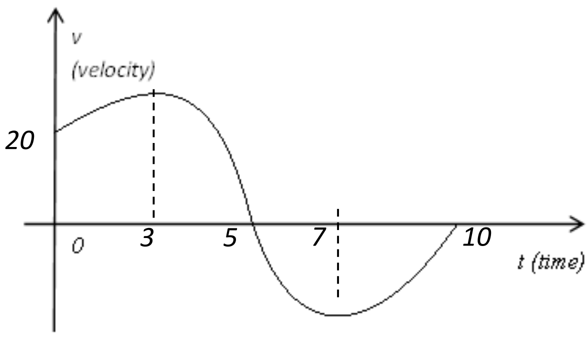
II. How many bass are left after 3 ½ years?

III. What is the rate of change of bass per month when t= 56?

IV. How long is it before there are 1000 bass, in years?

Motion

6.



NOT TO SCALE

I. What is the initial velocity of the particle?

II. When is the velocity of the particle equal to zero?

III. When is the acceleration of the particle equal to zero?

7. A particle moves along a straight line so that its distance x metres from a fixed point is given by

$$x = 4 - 3t + 9\ln(t + 1)$$

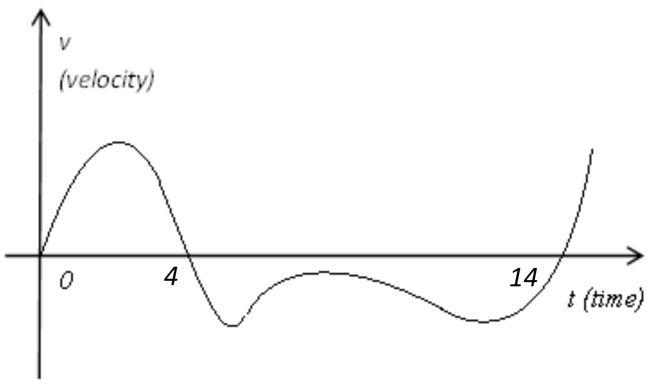
Where the time t is measured in seconds.

I. What is the position of the particle when t = 0?

II. Find expressions for the velocity and acceleration of the particle at time t

III. Find the time t when velocity of the particle is zero

8.



NOT TO SCALE

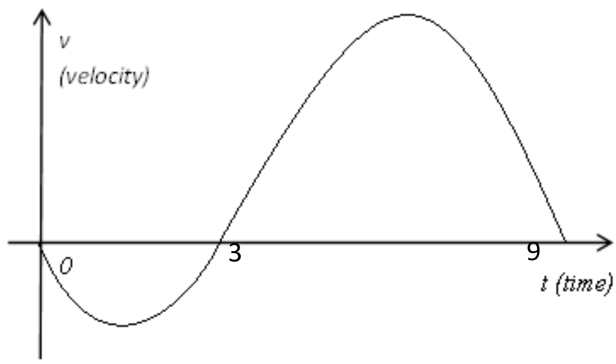
A particle is observed as it moves in a straight line in the period between t=0 and t=14. Its velocity v at time t is shown on the graph above.

I. Mark and clearly label with the letter Z the times when the acceleration of the particle is zero

II. Mark and clearly label with the letter G, the time when the acceleration is the greatest

III. There are three occasions when the particle is at rest, i.e t=0, t=4 and t=14. Indicate at which of these occasions, the particle is furthest from its original position, giving reasons for your answer.

9. **NOT TO SCALE**



A stick draws a line in the sand from the shore line in a straight line. Its velocity v at time t is shown on the graph above according to the equation

$$v = -2t^3 + 6t^2 - 8t$$

When t=0, the stick is 3cm from the shore line.

I. Find an expression for x, the distance of the tip of the stick from the shore line, as a function of t

II. What feature will the graph of x as a function of t have at t=1 and t=8?

III. The stick displaces 75g of sand per centimetre travelled. How much sand is displaced between t=0 and t=2?

Miscellaneous Algebra

1. Solve the pair of simultaneous equations:

$$2x + 4y = 9$$

$$x - y = 3$$

2. Factorize $x^2 + 7x - 30$.

3. Find those values of x which satisfy the inequality $4 - 5x > 14$

4. Rationalise the denominator of $\frac{2}{\sqrt{10}+2}$

5. Solve $3(x + 2) = \frac{x}{2} + 4$